Test 2

Proportion, Functions, Relations & Transformations
Semester One 2019
Year 11 Mathematics Methods
Calculator Assumed

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Name:	Sol
Teacher:	

Date: Friday 12th April 7.45am

You may have a formula sheet and 1 page (1 side) of notes for this test.

Total_____ / 41   Total Marks: 41   Time: 45 Minutes

Question 1   **(3 marks)**
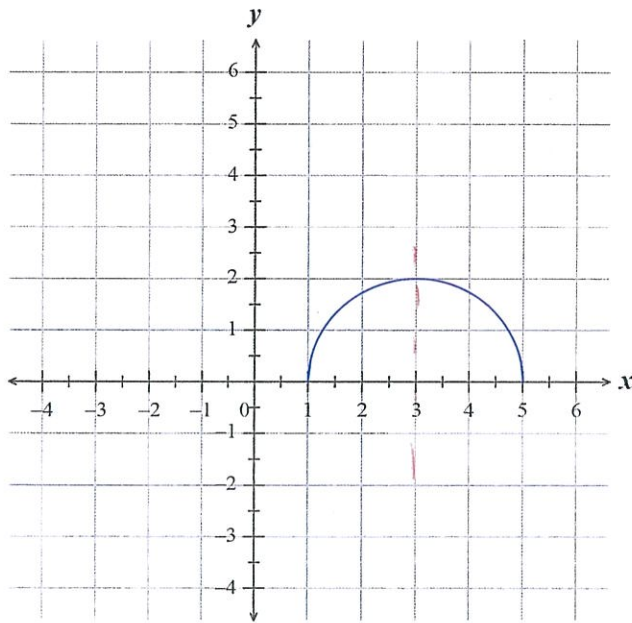
State whether the following relations are functions.
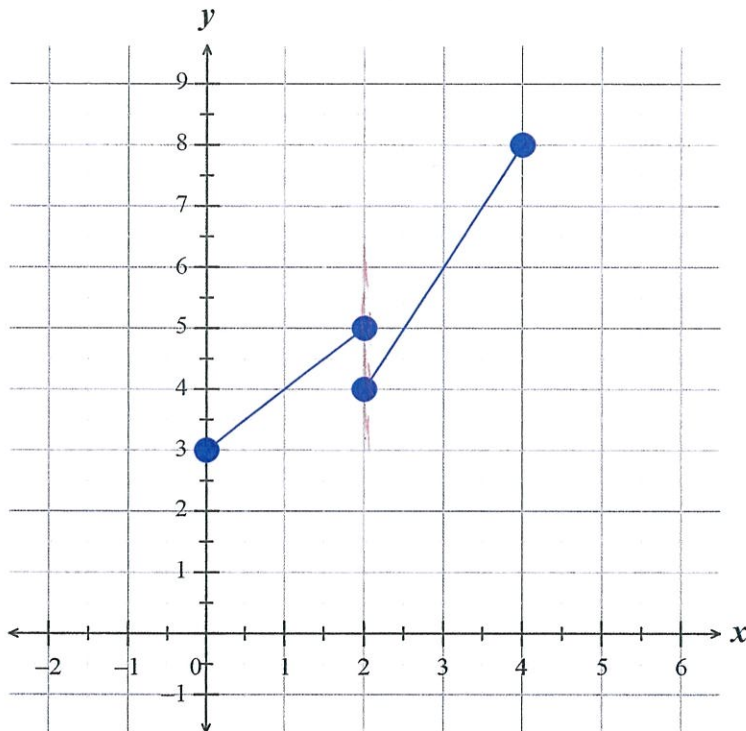
a) {(0, 0), (1, 1), (1, -1), (4, 2), (9, 3)}   No / Relation ✓

b)

Yes / Function ✓

c)

No / Relation ✓

Question 2 **(4 marks)**

Given that y is directly proportional to the square of x. When $y = 12$, $x = 4$, find

   a) the constant of variation **(2 marks)**

$$y \propto x^2$$

$$y = kx^2$$

$$12 = k(4^2)$$

$$12 = 16k \quad \checkmark$$

$$\frac{12}{16} = k \quad \checkmark$$

$$\frac{3}{4} = k$$

   b) the value(s) of x when y = 27 **(2 marks)**

$$27 = \frac{3}{4}x^2$$

$$\frac{\overset{9}{\cancel{27}} \times 4}{\cancel{3}} = x^2$$

$$36 = x^2 \quad \checkmark$$

$$\pm 6 = x \quad \checkmark \quad (\text{for both solutions})$$

Question 3   **(8 marks)**

(a) Find the radius and the coordinate of the centre of the circle with equation
$x^2 + y^2 - 4x - 6y - 3 = 0$. Show your working.   (3 marks)

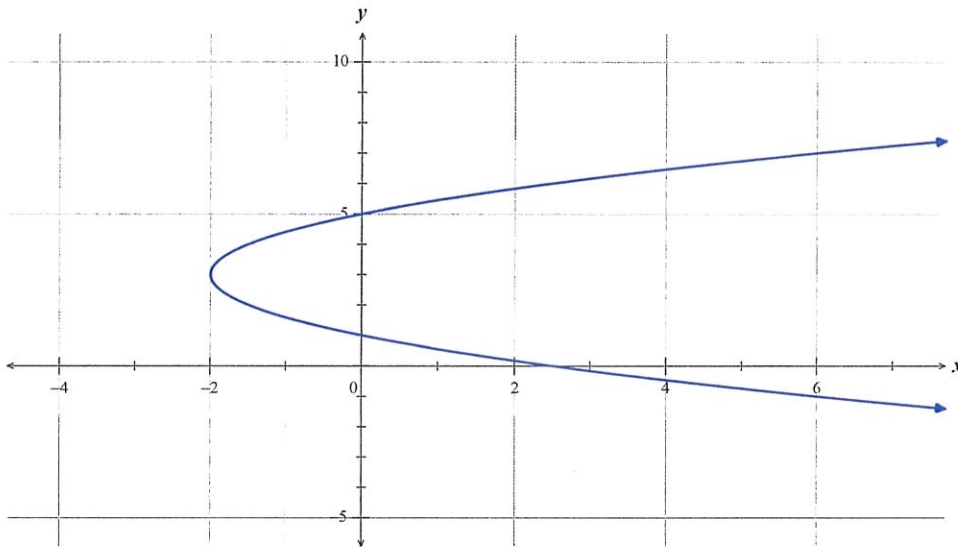$$x^2 - 4x + 4 - 4 + y^2 - 6y + 9 - 9 - 3 = 0$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 3 + 9 + 4$$

$$(x - 2)^2 + (y - 3)^2 = 16 ✓$$

center $(2, 3)$ ✓

radius 4 ✓

(b) The variables x and y are related as demonstrated by this graph.

i)   Determine the equation of the graph above.   (3 marks)

$$y^2 = x$$

$$(y - 3)^2 = a^2 (x + 2)$$ ✓ ✓

Sub $(0, 1)$ ✓

$$(1 - 3)^2 = a^2 (2)$$

$$4 = 2a^2$$

$$2 = a^2$$ ✓ $(y - 3)^2 = 2(x + 2)$

ii)   State the domain.   (1 mark)

$$x \geqslant -2$$ ✓

OR $[-2, \infty)$

c) From **(a)** and **(b)**, what features of their graphs clearly indicate that x is not a function
of y?   (1 mark)

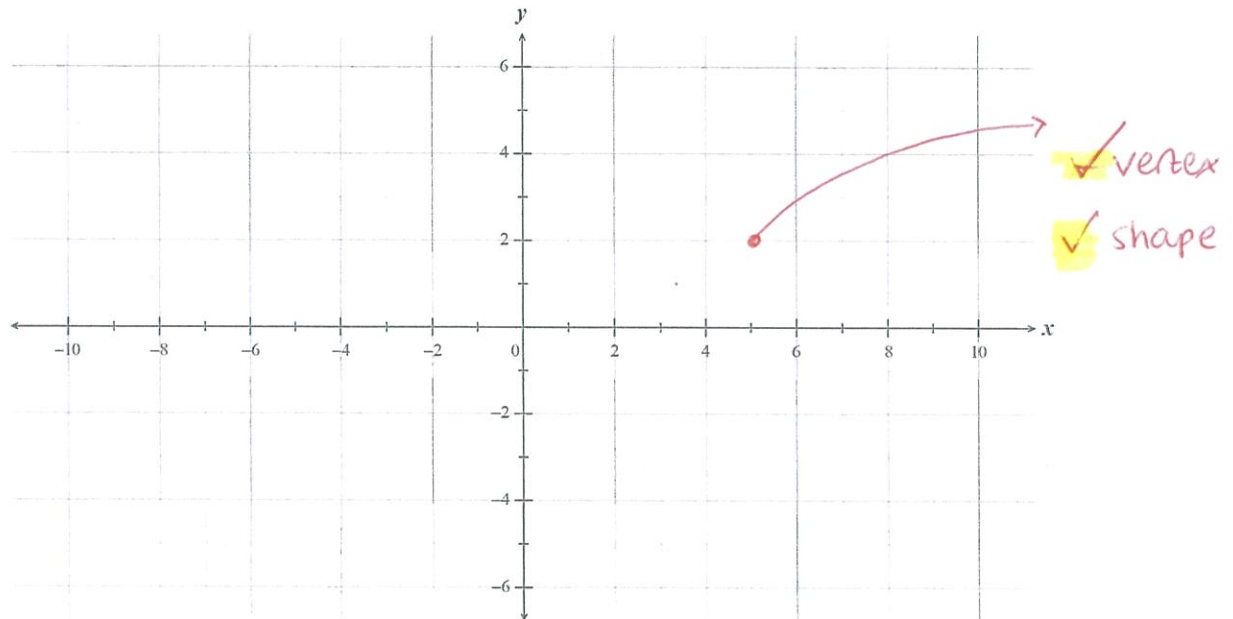There are two y-values for the same x-value. ✓

Question 4 (6 marks)

The function $f(x) = \sqrt{x}$ is transformed into $g(x) = k\sqrt{(ax+b)} + c$ by the following sequence of transformations.

(a) Sketch the following transformation of $f(x)$.

‘A translation 5 units in the positive x-axis followed by a translation of 2 units in the positive y-axis.’ (2 marks)

✓ vertex

✓ shape

(b) Determine the equations of the resulting function $g(x)$.

i) A translation 3 units in the direction of the negative y-axis followed by a reflection about the x-axis. (2 marks)

$$g(x) = -\sqrt{x} + 3$$

✓ ✓

ii) A dilation parallel to the positive x-axis of factor 2 followed by a translation 4 units in the direction of the positive x-axis (2 marks)

$$g(x) = \sqrt{\tfrac{1}{2}(x-4)}$$

✓ ✓

OR

$$g(x) = \sqrt{\tfrac{1}{2}x - 2}$$

✓ ✓

Question 5   **(9 marks)**

Consider the functions f and g where $f(x) = ax^2 + bx + c$ and $g(x) = f(2x + 3)$.

a) Given $f(-2) = 0, f(5) = 0$ and $f(2) = 3$, determine the rule for $f(x)$.   (3 marks)

$$f(x) = a(x+2)(x-5)$$

Sub $(2,3)$ OR

$$3 = a(4)(-3)$$
$$3 = -12a$$ $$-0.25x^2 + \tfrac{3}{4}x + \tfrac{5}{2}$$
$$-\tfrac{1}{4} = a$$ ✓ $$-\tfrac{1}{4}x^2 + \tfrac{3}{4}x + \tfrac{5}{2}$$

$$f(x) = -\tfrac{1}{4}(x+2)(x-5)$$ ✓

b) Express the rule for $g(x)$ as a polynomial.   (3 marks)

$$f(2x+3) = -\tfrac{1}{4}((2x+3)+2)((2x+3)-5)$$ ✓

$$= -\tfrac{1}{4}(2x+5)(2x-2)$$ ✓

$$= -\tfrac{1}{4}(4x^2 + 6x - 10)$$

$$= -x^2 - \tfrac{3}{2}x + \tfrac{5}{2}$$ ✓

c) The coordinate $(1,3)$ lies on $f(x)$. Determine the coordinate for $f(x) - 4$.   (1 mark)

$$(1, -1)$$ ✓

d) Describe the sequence of transformations that would transform $f(x)$ to $g(x)$.

                                                                (2 marks)

→ translate   3 units left ✓
→ dilate horizontally by s.f $\tfrac{1}{2}$ ✓   (-1 mark for incorrect order)

OR

$$f(2(x + \tfrac{3}{2}))$$

→ dilate horizontally by s.f $\tfrac{1}{2}$ ✓
→ translate $\tfrac{3}{2}$ units left. ✓

Question 6   **(4 marks)**

The time (t) in hours required to construct a retaining wall varies inversely to the number of workers (w) being employed. An engineer estimates that it will take 8 workers 180 hours to construct a retaining wall. [Assume that all workers work at the same rate.]

a) If the retaining wall must be constructed in 150 hours, how many extra workers will need to be employed?   (3 marks)

$$t \propto \frac{1}{w}$$

$$t = \frac{k}{w}$$

$$k = 8 \times 180$$

$$= 1440 \quad \checkmark$$

$$150 = \frac{1440}{w}$$

$$w = 9.6$$

$$w \approx 10 \quad \checkmark$$

∴ He will need to hire 2 more workers. ✓

b) If only 6 workers are available, how long will they take to construct this wall?   (1 mark)

$$t = \frac{1440}{6}$$

$$= 240 \text{ hours} \quad \checkmark$$

Question 7   **(7 marks)**

(a) Express $f(x) = \frac{6x-15}{x-3}$ into the form $f(x) = \frac{a}{x-h} + k$.   (2 marks)

without classpad

$$\frac{6(x-3) + 3}{x-3}$$

$$f(x) = \boxed{6} + \boxed{\frac{3}{x-3}}$$ ✓

OR

$$x-3 \overline{\smash{\big)}\, 6x - 15} \quad \overset{6}{}$$
$$\underline{6x - 18}$$
$$0 \;\; +3$$

with a classpad

$$F(x) = \frac{a}{x-3} + \boxed{6}$$ ✓

sub

$(0,5)$

$$\underset{-6}{5} = \underset{-3}{\frac{a}{}} + \underset{-6}{6}$$

$$-1 = \frac{a}{-3}$$

$$\boxed{3 = a}$$

$$f(x) = \boxed{\frac{3}{x-3}} + 6$$ ✓

(b) Determine the coordinate of the x-intercept.   (1 mark)

$$(2.5, 0)$$ ✓

(c) State the asymptotes of $f(x)$.   (2 marks)

vertical asymptote $x = 3$ ✓

horizontal asymptote $y = 6$ ✓

(d) Hence, determine the range of $f(x)$.   (2 marks)

$$y \in \mathbb{R}, \quad y \neq 6$$ ✓✓

OR

$$(-\infty, 6) \cup (6, +\infty)$$ ✓✓

END OF TEST